The values of the Nusselt number obtained in this way for structurally viscous liquids and water-glycerol solutions are given in Fig. 3. For comparison with formula (1) the values of the Nusselt number for structurally viscous liquids are multiplied by $\chi^{1/3}$, where

$$\chi = (1 + \Theta/\varphi_0\tau_w)/(1 + 0.8\,\Theta/\varphi_0\tau_w).$$

The experimental points lie satisfactorily close to a line, thus conforming the validity of formula (1). The maximum deviation of the experimental data from the averaging line does not exceed ±8%.

NOTATION

Nu_0 is the Nusselt number for laminar flow of Newtonian liquid; Nu is the Nusselt number for structurally viscous liquids; Pe_0 is the Peclet number determined from temperature at entrance to channel; Re_0 is the Reynolds number determined from zero fluidity ($\tau_w \to 0$) and temperature of liquid at entrance; $\chi = (1 + \Theta/\varphi_0\tau_w)/(1 + 0.8\Theta/\varphi_0/\tau_w)$ is a coefficient which takes into account the structurally viscous properties of a liquid with a linear fluidity law; Θ is the coefficient of structural instability of the liquid; φ_0 is the zero fluidity of liquid; τ_w is the tangential shear stress on wall.

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HEAT TRANSFER IN THE RADIATIVE HEATING AND DRYING OF GRANULAR MATERIAL IN A FLUIDIZED BED

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We consider the case where all the heat supplied to the fluidized bed by radiation through the upper boundary is carried off by the fluidizing medium (gas). The propagation of heat into the bed depends on the effective longitudinal thermal conductivity λ_{eff}. Neglecting longitudinal conductive heat transfer by the gas and regarding the bed as ideally fluidized and consisting of smooth spherical particles of equal diameter we write the Fourier-Kirchhoff equation for unit elementary volume:

$$\lambda_{eff}\frac{d^2\Theta}{dh^2} = \frac{6\alpha(1-\varepsilon)}{d}(\Theta - t), \tag{1}$$

$$\gamma_g c_g w_g \frac{dt}{dh} = \frac{6\alpha(1-\varepsilon)}{d}(\Theta - t). \tag{2}$$

Introducing the dimensionless height y = h/H of the bed and using the symbols $a = \lambda_{eff}/H$, $b = 6\alpha(1-\varepsilon)H/d\gamma_g c_g w_g$, $g = b\gamma_g c_g w_g = 6\alpha(1-\varepsilon)H/d$, we obtain

$$a\frac{d^2\Theta}{dy^2} - g(\Theta - t) = 0, \tag{3}$$

$$\frac{dt}{dy} - b(\Theta - t) = 0. \tag{4}$$

Expressing Θ from (4) and substituting in (3) we obtain after transformations

$$\frac{d^3t}{dy^3} + b\frac{d^2t}{dy^2} - \frac{g}{a}\frac{dt}{dy} = 0. \tag{5}$$

The general form of the solution of Eq. (5) is

$$t = C_1 \exp\left[-\frac{b}{2}(1-\Gamma)y\right] + C_2 \exp\left[-\frac{b}{2}(1+\Gamma)y\right] + C_3, \tag{6}$$

where

$$\Gamma = \sqrt{1 + 4g/ab^2}.$$

Using Eq. (4), we obtain an expression for

$$\Theta = C_1\frac{1+\Gamma}{2}\exp\left[-\frac{b}{2}(1-\Gamma)y\right] + C_2\frac{1-\Gamma}{2}\exp\left[-\frac{b}{2}(1+\Gamma)y\right] + C_3. \tag{7}$$

The boundary conditions are obtained from the following considerations. Since the absorption of radiation terminates in a thin upper zone of the bed [1, 2] we can assume that the radiative heat flux is delivered to the upper cross section of the bed; the heat is carried downward by the effective thermal conductivity of the bed. Hence, when y = 1,

$$a\frac{d\Theta}{dy} = q_{rad}. \tag{8}$$

There is no heat flux through the lower boundary of the bed and, hence, for y = 0

$$a\frac{d\Theta}{dy} = 0. \tag{9}$$

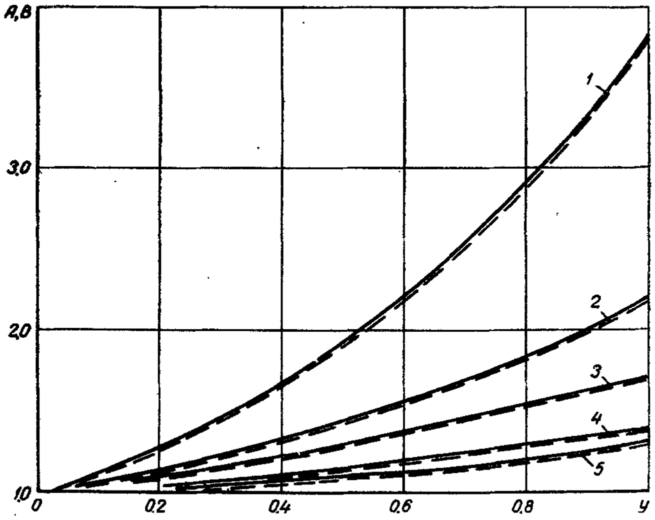
In addition, when y = 0, $t = t_1$.

Using the boundary conditions we finally obtain expressions for the gas temperature

$$t = t_1 + q_{rad} b \exp\left(-\frac{b}{2}y\right)\operatorname{sh}\left(\frac{b}{2}\Gamma y\right) \times \left[g\exp\left(-\frac{b}{2}\right)\operatorname{sh}\left(\frac{b}{2}\Gamma\right)\right]^{-1} \tag{10}$$

and for the temperature of the material

$$\Theta = t_1 + \left\{q_{rad} b \exp\left(-\frac{b}{2}y\right)\left[\operatorname{sh}\left(\frac{b}{2}\Gamma y\right) + \Gamma\operatorname{ch}\left(\frac{b}{2}\Gamma y\right)\right]\right\} \times \left[2g\exp\left(-\frac{b}{2}\right)\operatorname{sh}\left(\frac{b}{2}\Gamma\right)\right]^{-1}. \tag{11}$$



Plot of $A = (\Theta - t_1)/(\Theta_1 - t_1)$ (solid lines) and $B = (t - t_1)/(\Theta_1 - t_1)$ (broken lines) against height y of bed (from data of [3]) for w_g, m/sec: 1) 0.86; 2) 1.07; 3) 1.22; 3) 1.22; 4) 1.53; 5) 2.06.

When y = 0,

$$t = t_1, \quad \Theta = t_1 + q_{\mathrm{rad}} b\Gamma \Big/ 2g \exp\left(-\frac{b}{2}\right) \operatorname{sh}\left(\frac{b}{2}\Gamma\right). \quad (12)$$

When y = 1,

$$t = t_1 + q_{\mathrm{rad}} b/g, \quad (13)$$

$$\Theta = t_1 + \frac{q_{\mathrm{rad}} b}{2g \operatorname{sh}(b\Gamma/2)} \left[\operatorname{sh}\left(\frac{b}{2}\Gamma\right) + \Gamma \operatorname{ch}\left(\frac{b}{2}\Gamma\right)\right]. \quad (14)$$

The difference in the temperatures of the particles and gas is expressed as

$$\Theta - t = \frac{q_{\mathrm{rad}} b \exp(-by/2)}{2g \exp(-by/2) \operatorname{sh}(b\Gamma/2)} \times$$
$$\times \left[\Gamma \operatorname{ch}\left(\frac{b}{2}\Gamma y\right) - \operatorname{sh}\left(\frac{b}{2}\Gamma y\right)\right]. \quad (15)$$

The mean integral difference in the temperatures of the particles and gas throughout the bed can be determined by integrating expression (15):

$$(\Theta - t)_{\mathrm{cr}} = \int_0^1 (\Theta - t)\, dy = q_{\mathrm{rad}}/g. \quad (16)$$

Thus, the mean difference in temperatures of the particles and gas does not depend on the rate of mixing of the bed.

To evaluate the distribution of the particle and gas temperatures over the height of the bed we introduce the concepts of relative particle temperature

$$A = \frac{\Theta - t_1}{\Theta_1 - t_1} =$$
$$= \left\{\exp\left(-\frac{b}{2}y\right)\left[\operatorname{sh}\left(\frac{b}{2}\Gamma y\right) + \Gamma \operatorname{ch}\left(\frac{b}{2}\Gamma y\right)\right]\right\} \Big/ \Gamma \quad (17)$$

and relative gas temperature

$$B = \frac{t - t_1}{\Theta_1 - t_1} = \left[2 \operatorname{sh}\left(\frac{b}{2}\Gamma y\right) \exp\left(-\frac{b}{2}y\right)\right] \Big/ \Gamma. \quad (18)$$

The values of A and B are independent of the radiant heat flux q_{rad}.

The data for the longitudinal thermal conductivity of a fluidized bed of sand, given in [3], were used for a sample calculation. The coefficient of heat transfer between the particles and gas was calculated by using the expression [4]

$$\mathrm{Nu} = 0.316\, \mathrm{Re}^{0.8}. \quad (19)$$

The results of the calculation are given in Fig. 1.

The figure shows that at low gas filter velocities there is a considerable change in the temperatures of the material and gas over the height of the bed, and only at high filter velocities is this change small. Thus, the increase in λ_{eff} significantly affects the temperature distribution in the bed.

The experimentally observed [2] uniform temperature of the bed over the height (below the lower active zone of heat transfer) is presumably due to the higher values of the longitudinal thermal conductivity of smooth spherical particles in comparison with sand. In the experiments there were low fluidization numbers at which equation (19) does not hold. It may be that in fact there are higher heat transfer coefficients than those calculated from Eq. (19). In addition, owing to the small heat fluxes q_{rad} used the difference $\Theta_1 - t_1$ was low and, hence, within the limits of accuracy of temperature measurement in the experiments there were no temperature changes over the height of the bed except in the lower zone of the heat transfer of the particles and gas. A more accurate treatment of this question will require further experimental investigations. Having obtained the experimental temperature profile over the height of the bed for the particles and gas it is possible, on the basis of Eqs. (17) and (18), to use an electronic analog computer to select a value of λ_{eff} which satisfies the experimental data.

We consider the temperature distribution in the bed in the case of drying with heat supplied by radiation (we confine ourselves to a consideration of only the constant-rate period). The temperature of the particles in the bed can be regarded as constant and equal to the wet-bulb temperature $\Theta = t_M$ = const, and the heat balance equation is written as

$$\frac{dt}{dy} = b(t - \Theta) + k, \quad (20)$$

where $k = rmH/w_g c_g \gamma_g$. The value of m can be determined from the over-all heat balance of the bed:

$$m = [q_{\mathrm{rad}} + w_g c_g \gamma_g (t_1 - t_2)]/rH(1 - \varepsilon). \quad (21)$$

Solving Eq. (29) we obtain an expression for the gas temperature distribution over the height of the bed in the first drying period

$$t = [t_1 - \Theta + rmd/6\alpha(1 - \varepsilon)] \exp(by) - rmd/6\alpha(1 - \varepsilon) + \Theta. \quad (22)$$

The temperature of the gas on emergence from the bed is

$$t_2 = [t_1 - \Theta + rmd/6\alpha(1 - \varepsilon)] \exp b - rmd/6\alpha(1 - \varepsilon) + \Theta. \quad (23)$$

Equations (21) and (23) can be used to determine the gas temperature on emergence from the bed and the evaporation rate m or to determine the required heat flux q_{rad} for a prescribed evaporation rate.

NOTATION

Θ and t are the temperatures of the material and gas, respectively; λ_{eff} is the effective longitudinal thermal conductivity; α is the coefficient of heat transfer between particles and gas; h and H are the variable and total height of the bed, respectively; ε is the voidage of the bed; d is the particle diameter; w_g, c_g, and γ_g are the blowing velocity, specific heat, and density of the gas; q_{rad} is the radiative heat flux delivered to the bed; m is the rate of drying of a unit volume of bed; r is the heat of vaporization; t_M is the wet-bulb temperature. Subscripts 1 and 2 denote bottom and top cross sections of the bed.

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